

ДИНАМИКА ПЛАЗМЫ

УДК 533.9

DIFFUSION IN PLASMA: THE HALL EFFECT,
COMPOSITIONAL WAVES, AND CHEMICAL
SPOTS

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We consider diffusion caused by a combined influence of the electric current and Hall effect, and argue that such diffusion can form inhomogeneities of a chemical composition in plasma. The considered mechanism can be responsible for a formation of element spots in laboratory and astrophysical plasmas. This current-driven diffusion can be accompanied by propagation of a particular type of waves in which the impurity number density oscillates alone. These compositional waves exist if the magnetic pressure in plasma is much greater than the gas pressure.

1 INTRODUCTION

Often laboratory and astrophysical plasmas are multicomponent, and diffusion plays an important role in many phenomena in such plasmas. For instance, diffusion can be responsible for the formation of chemical inhomogeneities which influence emission, heat transport, conductivity, etc (see, e. g., [1–3]). In thermonuclear fusion experiments, the source of trace elements is usually the chamber walls, and diffusion determines the penetration depth of these elements and their distribution in plasma (see, e. g., [4–6]). Even a small admixture of heavy ions increases drastically radiative losses of plasma and changes its thermal properties. In astrophysical conditions, diffusion leads to the formation of element spots detected on the surface of many stars (see, e. g., [7–9]). Usually, diffusion in astrophysical bodies is influenced by a number of factors such as gravity, radiative force, magnetic field, temperature gradient, etc. (see, e. g., [10]). Under such conditions, diffusion processes may exhibit some rather unexpected properties that still have not been studied in laboratories.

Diffusion in plasma can differ qualitatively from that in neutral gases because of the presence of electrons and electric currents. A mean motion of electrons caused by electric currents provides an additional internal force that results in diffusion of trace elements (see, e. g., [11]). One more important contribution of electrons in diffusion is relevant to the Hall effect. The magnetic field can magnetize the charged particles that leads to the anisotropic transport. In the case of electron transport, such anisotropy is characterized by the Hall parameter, $x_e = \omega_{Be}\tau_e$, where $\omega_{Be} = eB/m_e c$ is the gyrofrequency of electrons and τ_e is their relaxation time, B is the magnetic field. In a hydrogen plasma, $\tau_e = 3\sqrt{m_e}(k_b T)^{3/2}/4\sqrt{2}\pi e^4 n_e \Lambda$ (see, e. g., [10]) where n_e and T are the number density of electrons and their temperature, respectively, Λ is the Coulomb logarithm. At $x_e \geq 1$, the

rates of diffusion along and across the magnetic field become different and, in general, diffusion can lead to the inhomogeneous distribution of elements.

In this paper, we consider one more diffusion process that can lead to formation of chemical inhomogeneities in plasma. This process is caused by the combined influence of the electric currents and the Hall effect. Using a simple model, we show that the interaction of the electric current with trace elements leads to their diffusion in the direction perpendicular to both the electric current and the magnetic field. This type of diffusion can alter the distribution of chemical elements in plasma and contribute to the formation of chemical spots even if the magnetic field is relatively weak and does not magnetize electrons ($x_e \ll 1$). We also argue that the current-driven diffusion in combination with the Hall effect can be the reason of the particular type of modes in which the number density of a trace element oscillates alone.

2 BASIC EQUATIONS AND DIFFUSION COEFFICIENTS

Consider plasma with the magnetic field parallel to the axis z , $\mathbf{B} = B\mathbf{e}_z$, where (s, φ, z) are cylindrical coordinates and $(\mathbf{e}_s, \mathbf{e}_\varphi, \mathbf{e}_z)$ are the corresponding unit vectors, respectively. We assume that plasma is cylindrical and the magnetic field depends on the cylindrical radius alone, $B = B(s)$. Then, the electric current is given by

$$j_\varphi = -(c/4\pi)\partial B/\partial s. \quad (1)$$

We suppose that $j_\varphi \rightarrow 0$ at large s and, hence, $B \rightarrow B_0 = \text{const}$ at $s \rightarrow \infty$. Note that the dependence $B(s)$ can not be an arbitrary function of s because, generally, the cylindrical magnetic configurations are unstable if $B(s)$ increases with s or decreases sufficiently slowly (see, e. g., [12–14]). In astrophysical bodies, the magnetic field usually has a more complex topology than our simple configuration. However, this model describes correctly the main qualitative features of current-driven diffusion. In some cases, this model can even mimic the magnetic field in certain regions of a star. For example, the field near the magnetic pole has a topology very close to our model (1) (see, e. g., [15]).

We assume that plasma consists of electrons e , protons p , and a small admixture of heavy ions i . The number density of species i is small and it does not influence dynamics of plasma. Therefore, this species can be treated as trace particles that interact only with a background hydrogen plasma. The partial momentum equations in fully ionized multicomponent plasma has been considered by a number of authors (see, e. g., [16, 17]). These equations can be obtained by multiplying the Boltzmann kinetic equation for each species by its velocity and integrating over it. The momentum equation for particles α ($\alpha = e, p, i$) reads

$$m_\alpha n_\alpha \left[\dot{\mathbf{V}}_\alpha + (\mathbf{V}_\alpha \cdot \nabla) \mathbf{V}_\alpha \right] = -\nabla p_\alpha + n_\alpha \mathbf{F}_\alpha + eZ_\alpha n_\alpha \left(\mathbf{E} + \frac{\mathbf{V}_\alpha}{c} \times \mathbf{B} \right) + \mathbf{R}_\alpha, \quad (2)$$

the dot denotes the partial time derivative. Here, m_α and Z_α are the mass and the charge number of particles α , n_α and p_α are their number density and pressure, respectively, \mathbf{V}_α is the mean velocity, \mathbf{F}_α is an external force acting on the particles α ; \mathbf{E} and \mathbf{B} are the electric and magnetic fields, respectively; \mathbf{R}_α is the internal friction force caused by the

collisions of particles α with other sorts of particles. Since \mathbf{R}_α is the internal force, the sum of \mathbf{R}_α over α is zero in accordance with the Newton's third law. Usually, the force \mathbf{F}_α is the sum of the gravitational and radiation force. Below we will neglect it.

If there are no mean hydrodynamic velocity and only diffusive velocities of trace elements are non-vanishing, the partial momentum equation for particles α reads

$$-\nabla p_\alpha + Z_\alpha e n_\alpha \left(\mathbf{E} + \frac{\mathbf{V}_\alpha}{c} \times \mathbf{B} \right) + \mathbf{R}_\alpha = 0. \quad (3)$$

In Eq. (3) for the trace particles i , we can represent the friction forces \mathbf{R}_i as $\mathbf{R}_i = \mathbf{R}_{ie} + \mathbf{R}_{ip}$, where the force \mathbf{R}_{ie} is caused by scattering of the ions i on the electrons and \mathbf{R}_{ip} by scattering on the protons.

If n_i is small compared to the number density of protons n_p , \mathbf{R}_{ie} is given approximately by

$$\mathbf{R}_{ie} = -\frac{Z_i^2 n_i}{n_p} \mathbf{R}_e \quad (4)$$

where \mathbf{R}_e is the force acting on the electron gas (see, e. g., [18]). Since $n_i \ll n_p$, \mathbf{R}_e is determined mainly by scattering of electrons on protons but scattering on ions i gives a small contribution. Therefore, we can use for \mathbf{R}_e the expression for one component hydrogen plasma calculated by Braginskii [16]. In our model of a cylindrical plasma configuration, this expression reads

$$\mathbf{R}_e = -\alpha_\perp \mathbf{u} + \alpha_\wedge \mathbf{b} \times \mathbf{u} - \beta_\perp^{uT} \nabla T - \beta_\wedge^{uT} \mathbf{b} \times \nabla T, \quad (5)$$

where $\mathbf{u} = -\mathbf{j}/en$ is the difference between the mean velocities of electrons and protons; α_\perp , α_\wedge , β_\perp^{uT} , and β_\wedge^{uT} are the coefficients calculated in [16]; $\mathbf{b} = \mathbf{B}/B$. The first two terms on the r.h.s. of Eq.(5) describe the standard friction force caused by a relative motion of the electron and proton gases. The last two terms on the r.h.s. of Eq.(5) represent the so-called thermoforce caused by a temperature gradient. This part of \mathbf{R}_e is responsible for thermodiffusion. For the sake of simplicity, we consider plasma with a uniform temperature, $\nabla T = 0$.

Taking into account that $\mathbf{u} = u\mathbf{e}_\varphi$ in our model and using coefficients α_\perp and α_\wedge calculated in [16], we obtain the following expressions for the cylindrical components of \mathbf{R}_{ie}

$$R_{ie\varphi} = Z_i^2 n_i \left(\frac{m_e}{\tau_e} \delta_1 u \right), \quad R_{ies} = Z_i^2 n_i \left(\frac{m_e}{\tau_e} \delta_4 u \right), \quad (6)$$

where

$$\begin{aligned} \delta_1 &= 1 - \delta_3^{-1} (1.84 + 6.42x^2), \quad \delta_4 = \delta_3^{-1} x (0.78 + 1.7x^2), \\ \delta_3 &= x^4 + 14.79x^2 + 3.77, \quad x = \omega_{Be} \tau_e. \end{aligned} \quad (7)$$

The force \mathbf{R}_{ip} consists of two parts as well, \mathbf{R}'_{ip} and \mathbf{R}''_{ip} , which are proportional to the relative velocity of ions i and protons and to the temperature gradient, respectively. The thermoforce is vanishing in our model. The friction force \mathbf{R}'_{ip} can be easily calculated in the most interesting case when the mass of a species i , m_i , is greater than the proton mass, m_p . In this case, \mathbf{R}'_{ip} is proportional to the relative velocity of heavy ions and the background plasma, $(\mathbf{V}_p - \mathbf{V}_i)$. Taking into account that the mean velocity of the

background plasma in our simplified model is assumed to be zero, the friction force can be represented as (see, e. g., [1, 17])

$$\mathbf{R}'_{ip} = \frac{0.42m_i n_i Z_i^2}{\tau_i} (-\mathbf{V}_i), \quad (8)$$

where $\tau_i = 3\sqrt{m_i}(k_B T)^{3/2}/4\sqrt{2\pi}e^4 n_p \Lambda$ and τ_i/Z_i^2 is the characteristic timescale of ion-proton scattering; we assume that Coulomb logarithms are the same for all types of scattering. Since the number density of trace particles is small, we can suppose in calculations $n_p \approx n_e = n$.

The momentum equation for the species i (see Eq.(3)) contains cylindrical components of the electric field, E_s and E_φ . These components can be determined from the momentum equations (3) for electrons and protons

$$-\nabla(n_e k_B T) - en_e \left(\mathbf{E} + \frac{\mathbf{u}}{c} \times \mathbf{B} \right) + \mathbf{R}_e = 0, \quad (9)$$

$$-\nabla(n_p k_B T) + en_p \mathbf{E} - \mathbf{R}_e + \mathbf{F}_p = 0. \quad (10)$$

Taking into account the condition of hydrostatic equilibrium and quasi-neutrality ($n_e \approx n_p$), we obtain the following expressions for the radial and azimuthal electric fields

$$E_s = -\frac{uB}{2c} - \frac{1}{e} \left(\frac{m_e u}{\tau_e} \delta_4 \right), \quad E_\varphi = -\frac{1}{e} \left(\frac{m_e u}{\tau_e} \delta_1 \right). \quad (11)$$

Substituting Eqs. (6), (8), and (11) into Eq. (3) for the trace particles i , we arrive to the expression for a diffusion velocity \mathbf{V}_i ,

$$\mathbf{V}_i = V_{is} \mathbf{e}_s + V_{i\varphi} \mathbf{e}_\varphi, \quad V_{is} = V_{ni} + V_B, \quad (12)$$

where

$$V_{ni} = -D \frac{d \ln n_i}{ds}, \quad V_B = D_B \frac{d \ln B}{ds}, \quad V_{i\varphi} = D_{B\varphi} \frac{dB}{ds}; \quad (13)$$

V_{ni} is the velocities of ordinary diffusion and V_B and $V_{i\varphi}$ are the radial and azimuthal diffusion velocities caused by the electric current. The corresponding diffusion coefficients are

$$\begin{aligned} D &= \frac{2.4c_i^2 \tau_i}{Z_i^2(1+q^2)}, \quad c_i^2 = \frac{k_B T}{m_i}, \quad q = \frac{2.4eB\tau_i}{Z_i m_i c}, \\ D_B &= \frac{2.4cB\sqrt{m_e/m_i}}{4\pi en(1+q^2)} \left[\left(1 - \frac{1}{Z_i}\right) (\delta_4 + q\delta_1) - \frac{x}{2Z_i} \right], \\ D_{B\varphi} &= \frac{2.4c\sqrt{m_e/m_i}}{4\pi en(1+q^2)} \left[\left(1 - \frac{1}{Z_i}\right) (\delta_1 - q\delta_4) + \frac{qx}{2Z_i} \right]. \end{aligned} \quad (14)$$

Eqs. (12)–(14) describe the drift of ions i under a combined influence of ∇n_i and \mathbf{j} .

If magnetic field is weak and $x \ll 1$, Eq. (14) yields

$$\begin{aligned} D &\approx \frac{2.4c_i^2 \tau_i}{Z_i^2}, \quad D_B \approx \frac{2.4c_A^2 \tau_i}{Z_i A_i} (0.2Z_i - 0.7), \\ D_{B\varphi} &\approx 1.2 \sqrt{\frac{m_e}{m_i}} \frac{c(Z_i - 1)}{4\pi en Z_i}. \end{aligned} \quad (15)$$

where $c_A^2 = B^2/(4\pi n m_p)$ [18]. The coefficient D is always positive but two other diffusion coefficients can be positive or negative depending on the parameters of plasma.

In the opposite case of a very strong magnetic field, $q \gg 1$, Eq. (14) yields

$$D \approx \frac{2.4c_i^2\tau_i}{Z_i^2q^2}, \quad D_B \approx -\frac{1.2c_A^2\tau_i}{Z_iA_iq^2}, \quad D_{B\varphi} \approx \frac{c}{8\pi en}. \quad (16)$$

In our model, diffusion in the radial direction is strongly suppressed because all sorts of particles are magnetized. For instance, the coefficient D (Eq. (16)) and the corresponding diffusion velocity V_{ni} which characterize the standard diffusion in the s -direction are $\approx q^2 \gg 1$ times smaller than those in the case of a weak magnetic field. The coefficient D_B is also approximately q^2 times smaller in a strong magnetic field. Note that D_B reaches saturation and do not depend on the field strength at $q \gg 1$. If the electric current is fixed ($dB/ds = \text{const}$), the radial diffusion velocity V_B caused by currents decreases $\propto 1/B$. As far as the azimuthal diffusion is concerned, the coefficient $D_{B\varphi}$ does not depend on the magnetic field in both cases, strong and weak magnetic fields. However, $D_{B\varphi}$ in a strong field is greater by a factor $\sim \sqrt{m_i/m_e}$.

3 ELEMENT SPOTS CAUSED BY ELECTRIC CURRENTS

It is generally believed that standard diffusion smoothes chemical inhomogeneities on a diffusion timescale $\sim L^2/D$ where L is the lengthscale of a non-uniformity. This is not the case, however, for diffusion given by Eq. (12). In this case, chemical inhomogeneities can exist during a much longer time than $\sim L^2/D$ because the equilibrium distribution is reached due to balance of two diffusion processes, standard ($\propto \nabla n_i$) and current-driven ($\propto dB/ds$) ones, which push ions in the opposite directions. As a result, $V_{is} = 0$ in the equilibrium state and this state can be maintained as long as the electric current exists.

Note that the radial velocity is vanishing in the equilibrium state but the azimuthal velocity is non-zero. It turns out that impurities rotate around the magnetic axis even if equilibrium is reached. The direction of rotation depends on the sign of dB/ds and is opposite to the electric current. Since electrons move in the same direction, heavy ions turn out to be carried along the flow of electrons. Different ions move with different velocities around the axis. If the magnetic field is weak ($x \ll 1$), the difference between different sorts of ions, $\Delta V_{i\varphi}$, is of the order of

$$\Delta V_{i\varphi} \sim \frac{c}{4\pi en} \sqrt{\frac{m_e}{m_i}} \frac{dB}{ds} \sim 3 \times 10^{-3} \frac{B_4}{n_{14} L_{10} A_i^{1/2}} \frac{\text{cm}}{\text{s}}, \quad (17)$$

where $B_4 = B/10^4$ G, $n_{14} = n/10^{14} \text{ cm}^{-3}$, and $L_{10} = L/10^{10} \text{ cm}$. Since different impurities rotate around the magnetic axis with different velocities, periods of such rotation are also different for different ions. The difference in periods can be estimated as

$$\Delta P = \frac{2\pi L}{\Delta V} \sim 10^6 \frac{L_{10}^2 n_{14} A_i^{1/2}}{B_4} \text{ yrs.} \quad (18)$$

If the distribution of impurities is non-axisymmetric then such diffusion in the azimuthal direction should lead to slow variations in the abundance peculiarities. Note that in the

case of a strong field ($q \gg 1$), all sorts of trace particles rotate around the axis with the same period that depends only on the number density and electric current.

The condition of hydrostatic equilibrium in our model is given by

$$-\nabla p + \mathbf{j} \times \mathbf{B}/c = 0, \quad (19)$$

where p and ρ are the pressure and density, respectively. Since the background plasma is hydrogen, $p \approx 2nk_B T$ where k_B is the Boltzmann constant. Integrating the s -component of Eq. (19) and taking into account that the temperature is constant in our model, we obtain

$$n = n_0 (1 + \beta_0^{-1} - \beta^{-1}), \quad (20)$$

where $\beta = 8\pi p_0/B^2$; (p_0, n_0, T_0, β_0) are the values of (p, n, T, β) at $s \rightarrow \infty$.

Consider the equilibrium distribution of trace elements in cylindrical plasma. In equilibrium, we have $V_{is} = 0$ and Eq. (12) yields

$$\frac{d \ln n_i}{ds} = \frac{D_B}{D} \frac{d \ln B}{ds}. \quad (21)$$

The term on the r.h.s. describes the effect of electric currents on the distribution of trace elements. Note that this type of diffusion is driven by the electric current rather than an inhomogeneity of the magnetic field. Occasionally, the conditions $dB/ds \neq 0$ and $j \neq 0$ are equivalent in our simplified model.

First we consider the case of a weak magnetic field with $x \ll 1$. Then, one has from Eq. (19)

$$\frac{d}{ds}(nk_B T) = -\frac{B}{8\pi} \frac{dB}{ds}. \quad (22)$$

Substituting Eq. (22) into Eq. (21) and integrating, we obtain

$$\frac{n_i}{n_{i0}} = \left(\frac{n}{n_0} \right)^\mu, \quad (23)$$

where

$$\mu = -2Z_i(0.2Z_i - 0.7) \quad (24)$$

and n_{i0} is the value of n_i at $s \rightarrow \infty$. Denoting the local abundance of the element i as $\gamma_i = n_i/n$ and taking into account Eq. (19), we have

$$\frac{\gamma_i}{\gamma_{i0}} = \left(\frac{n}{n_0} \right)^{\mu-1} = \left(1 + \frac{1}{\beta_0} - \frac{1}{\beta} \right)^{\mu-1}, \quad (25)$$

where $\gamma_{i0} = n_{i0}/n_0$. Local abundances turn out to be flexible to the field strength and, particularly, this concerns the ions with large charge numbers. If other mechanisms of diffusion are negligible and the distribution of elements is basically current-driven, then the exponent $(\mu - 1)$ can reach large negative values for elements with large Z_i and, hence, produce strong abundance anomalies. For instance, $(\mu - 1)$ is equal 1.16, -0.52 , and -2.04 for $Z_i = 2, 3$, and 4 , respectively. Note that $(\mu - 1)$ changes its sign as Z_i increases: $(\mu - 1) > 0$ if $Z_i = 2$ but $(\mu - 1) < 0$ for $Z_i \geq 3$. Therefore, elements with $Z_i \geq 3$ are in deficit ($\gamma_i < \gamma_{i0}$) in the region with a weak magnetic field ($B < B_0$) but, on the contrary, these elements should be overabundant in the region where the magnetic field is stronger than B_0 .

The distribution of the impurities can be substantially different if the magnetic field is strong and $q \gg 1$. Using the same procedure as in the case of a weak field, we obtain

$$\frac{\gamma_i}{\gamma_{i0}} = \left(\frac{n}{n_0}\right)^{Z_i-1} = \left(1 + \frac{1}{\beta_0} - \frac{1}{\beta}\right)^{Z_i-1}. \quad (26)$$

Therefore, all trace elements with $Z_i > 1$ are overabundant in the regions with the magnetic field weaker than B_0 . On the contrary, these elements are underabundant in the regions with a stronger magnetic field.

Note that calculating \mathbf{E} from Eqs. (9) and (10), we neglect the electric field generated by redistribution of heavy ions because the number density of such ions is small. This electric field will decrease formation of spots and can produce departures from the simple picture outlined in this section. However, these departures are basically small since $n_i \ll n$, and they begin to play an important role only if the electric field generated by the redistribution of impurities in the spot is comparable to \mathbf{E} . Using Eqs. (9) and (10), one can estimate that the influences of these electric fields becomes comparable if $Z_i n_i \sim n$ in the spot. This equation determines the impurity number density above which our consideration is unjustified.

4 COMPOSITIONAL WAVES

In our simplified model of plasma cylinder with the velocity given by Eq. (12), the continuity equation for trace ions i reads

$$\frac{\partial n_i}{\partial t} + \frac{1}{s} \frac{\partial}{\partial s} (s n_i V_{is}) + \frac{1}{s} \frac{\partial}{\partial \varphi} (n_i V_{i\varphi}) = 0. \quad (27)$$

Consider the behavior of small disturbances of the number density of trace ions by making use of a linear analysis of Eq. (27). In the basic (unperturbed) state, plasma is assumed to be in a diffusive equilibrium and, hence, the unperturbed impurity number density satisfies Eq. (21). Since the number density of impurity i is small, its influence on parameters in the basic state is negligible. For the sake of simplicity, we consider disturbances that do not depend on z . Denoting disturbances of the impurity number density by δn_i and linearizing Eq. (26), we obtain the equation governing the evolution of such small disturbances,

$$\begin{aligned} \frac{\partial \delta n_i}{\partial t} - \frac{1}{s} \frac{\partial}{\partial s} \left(s D \frac{\partial \delta n_i}{\partial s} - s \delta n_i \frac{D_B}{B} \frac{dB}{ds} \right) + \\ \frac{1}{s} \frac{\partial}{\partial \varphi} \left(\delta n_i D_{B\varphi} \frac{dB}{ds} \right) = 0. \end{aligned} \quad (28)$$

For the purpose of illustration, we consider only disturbances with the wavelengths shorter than the lengthscale of unperturbed quantities. In this case, we can use the so called local approximation for a consideration of linear waves and assume that small disturbances are $\propto \exp(-iks - iM\varphi)$ where k is the wavevector ($ks \gg 1$) and M is the azimuthal wavenumber. Since the basic state does not depend on time, δn_i can be represented as $\delta n_i \propto e^{i\omega t - iks - iM\varphi}$ where ω should be calculated from the dispersion equation. Substituting δn_i in such form into Eq. (28), we obtain the following dispersion equation

$$\begin{aligned} i\omega = -\omega_R + i\omega_I, \quad \omega_R = Dk^2, \quad \omega_I = \omega_S + \omega_\varphi, \\ \omega_S = kD_B \frac{d \ln B}{ds}, \quad \omega_\varphi = \frac{M}{s} B D_{B\varphi} \frac{d \ln B}{ds}. \end{aligned} \quad (29)$$

This dispersion equation describes spiral waves in which only the number density of impurities oscillates and, therefore, such waves can be called “compositional”. The quantity ω_R characterizes decay of these waves with the characteristic timescale $\sim (Dk^2)^{-1}$ typical for a standard diffusion. The frequency ω_I describes oscillations of impurities caused by the combined action of electric current and the Hall effect. Note that ω_I can be of any sign but ω_R is always positive. The frequency ω_S characterizes oscillations in the radial direction and ω_φ is in the azimuthal direction.

The compositional waves are aperiodic if $\omega_R > |\omega_I|$ and oscillatory if $|\omega_I| > \omega_R$. We consider the compositional waves in particular cases of weak ($x \ll 1$) and strong ($q \gg 1$) magnetic fields.

Weak magnetic field ($x \ll 1$). If $ks \gg M$ (radial waves), the condition $|\omega_I| > \omega_R$ in a weak field is equivalent to

$$c_A^2/c_s^2 > Z_i^{-1}|0.21Z_i - 0.71|^{-1}kL, \quad (30)$$

where $L = |d \ln B / ds|^{-1}$ and c_s is the sound speed, $c_s^2 = k_B T / m_p$. In the opposite case $M \gg ks$ (azimuthal waves), the compositional waves are oscillatory if

$$c_A^2/c_s^2 \gg x(ks/M)(kL). \quad (31)$$

Both conditions (30) and (31) require very strong magnetic field so the magnetic pressure is substantially greater than the gas pressure. The frequency of compositional waves is higher in the region where the magnetic field has a stronger gradient or, in other words, where the density of electric currents is greater. Note that different impurities oscillate with different frequencies.

Consider first the radial waves with $M = 0$. Substituting $M = 0$ into Eq. (29), we obtain the dispersion equation for such waves in the form

$$i\omega = -\omega_R + i\omega_B, \quad \omega_R = Dk^2, \quad \omega_B = kD_B \frac{d \ln B}{ds}. \quad (32)$$

This dispersion equation describes waves in which only the number density of trace particles oscillates and oscillations of n_i occur only in the radial direction. The order of magnitude estimate of ω_S yields

$$\omega_I \sim kc_A \frac{1}{Z_i A_i} \frac{c_A}{c_i} \frac{l_i}{L}, \quad (33)$$

where $l_i = c_i \tau_i$ is the mean free-path of ions i . Note that different impurities oscillate with different frequencies. Therefore, if there are several sorts of trace ions in plasma, the chemical structure should exhibit variations of local abundances under the influence of compositional waves.

The dispersion equation for non-axisymmetric waves with $M \gg ks$ reads in a weak field

$$i\omega = -\omega_R + i\omega_{B\varphi}, \quad \omega_{B\varphi} = \frac{M}{s} BD_{B\varphi} \frac{d \ln B}{ds}. \quad (34)$$

In non-axisymmetric waves, trace ions rotate around the cylindrical axis with the frequency ω_φ and decay slowly on the diffusion timescale $\sim \omega_R^{-1}$. The frequency of such waves is typically higher than that of the radial waves. One can estimate the ratio of these frequencies as

$$\frac{\omega_\varphi}{\omega_S} \sim \frac{BD_{B\varphi}}{D_B} \sim \frac{1}{A_i x} \frac{M}{ks}. \quad (35)$$

Since these estimates are justified only in the case of a weak magnetic field ($x \ll 1$), the period of non-axisymmetric waves is shorter for waves with $M > A_i x(ks)$. The ratio of diffusion timescale and period of non-axisymmetric waves is

$$\frac{\omega_\varphi}{\omega_R} \sim \frac{1}{x} \frac{c_A^2}{c_s^2} \frac{Z_i}{A_i} \frac{1}{kL} \quad (36)$$

and can be large. Therefore, azimuthal waves can be oscillatory as well.

Strong magnetic field ($q \gg 1$) In a strong magnetic field, the order of magnitude estimates of the characteristic frequencies are

$$\omega_S \approx \frac{k}{2q} \frac{j_\varphi}{en}, \quad \omega_\varphi \approx -\frac{M}{2s} \frac{j_\varphi}{en}. \quad (37)$$

Like the case of a weak field, the frequency of compositional waves is higher in the region where the density of the electric currents is greater. Oscillations of different trace ions occur with different frequencies in radial waves but azimuthal oscillations have the same frequency for different impurities. The frequency of azimuthal waves is higher than that of radial waves if

$$M \gg \frac{ks}{q}. \quad (38)$$

If the magnetic field is **so** strong that $q \gg 1$ than the azimuthal waves oscillate with a higher frequency than the radial ones even for not very large M . The condition that radial waves exists in a strong magnetic field, $|\omega_S| \gg \omega_R$, is given by

$$\frac{c_A^2}{c_s^2} > \frac{2}{Z_i} kL. \quad (39)$$

Similar to the case of a weak magnetic field, compositional waves occur in plasma only if the magnetic pressure is greater than the gas pressure. The analogous condition for azimuthal waves, $\omega_\varphi \gg \omega_R$, reads

$$\frac{c_A^2}{c_s^2} > \frac{1}{qZ_i} \frac{ks}{M} kL. \quad (40)$$

Note that this condition can be satisfied even if the magnetic pressure is smaller than the gas one but q and M are large.

5 CONCLUSIONS

We have considered diffusion of heavy ions under the influence of electric currents. Generally, the diffusion velocity in this case can be comparable to or even greater than that caused by other diffusion mechanisms. The current-driven diffusion can form chemical inhomogeneities even if the magnetic field is relatively weak whereas other diffusion mechanisms require a substantially stronger magnetic field.

The current-driven diffusion is relevant to the Hall effect and, therefore, it leads to a drift of ions in the direction perpendicular to both the magnetic field and the electric current. As a result, distribution of chemical elements in plasma depends essentially on the geometry of the magnetic fields and the electric current. Chemical inhomogeneities

can manifest themselves, for example, by emission in spectral lines and a non-uniform plasma temperature. Usually, diffusion processes play an important role in plasma if hydrodynamic motions are very slow. In some cases, however, chemical spots can be formed even in flows with a relatively large velocity but with some particular topology (for example, a rotating flow). This can occur usually in laminar flows. Unfortunately, such flows often are unstable in magnetized plasma. This is particularly concerned to the flows with a large Hall parameter since hydrodynamic motions in such plasma typically are unstable even in the presence of a weak shear (see, e. g., [19–21]). As a result, a formation of the chemical spots is unlikely if there are hydrodynamic motions even with a weak shear.

The current-driven diffusion in combination with other diffusion mechanisms can be important for the surface chemistry of various types of stars. The mechanism considered can operate in various astrophysical bodies where the electric currents are non-vanishing. As it was noted, the current-driven diffusion leads to a formation of chemical spots only if the star has quiescent surface layers. That is the case, for instance, for white dwarfs and neutron stars. Observations detect strong magnetic fields in many neutron stars and, likely, topology of these fields should be rather complex with spot-like structures at the surface. As it was shown in our study, such magnetic structures can be responsible for the formation of element spots at the surface. A spot-like distribution of chemical elements can be important for the emission spectra, diffusive nuclear burning (see, e. g., [22, 23]), etc. Evolution of neutron stars is very complicated, particularly, in binary systems (see, e. g., [24]) and, as a result, a surface chemistry can be complicated as well. Diffusion processes play an important role in this chemistry (see, e. g., [25, 26]) and can be the reason of chemical spots on the surface of these stars.

Certainly, this type of diffusion may play an important role in the surface chemistry of the so called Ap/Bp-stars. These stars have a strong magnetic field [7] that magnetizes the atmospheric plasma and produces a rapid Hall drift of electrons. Using Eq. (15), one can estimate the velocity of current-driven diffusion as

$$V_B \sim 1.1 \times 10^{-4} A_i^{-1/2} B_4^2 n_{15}^{-2} T_4^{3/2} \Lambda_{10} L_{B10}^{-1} \text{ cm/s}, \quad (41)$$

where $\Lambda_{10} = \Lambda/10$, $B_4 = B/10^4 \text{ G}$, and $L_{B10} = L_B/10^{10} \text{ cm}$. The velocity V_B turns out to be sensitive to the field ($\propto B^2$) and, therefore, diffusion in a weak magnetic field requires a longer time to reach equilibrium. Since $B_4 \sim 1$, $T_4 \sim 1$, and $L_{B10} \sim 1$ are more or less typical values for Ap/Bp stars one can estimate that the timescale of spot formation in the atmosphere is shorter than the lifetime of such stars. Therefore, the current-driven diffusion can contribute to the generation of chemical structures in these stars. The conditions in Ap/Bp stars are also suitable for the propagation of compositional waves and, likely, such waves can be the reason of variations in atmospheric abundances of these stars.

The considered mechanism can operate in laboratory plasma as well. For instance, plasma adiabatically compressed and heated in experiments with explosives can reach very high values of the temperatures $T \sim 10^7\text{--}10^8 \text{ K}$, number density $n \sim 10^{20}\text{--}10^{21} \text{ cm}^{-3}$, and magnetic field $B \sim 10^6 \text{ G}$. In multiple mirror experiments with the improved confinement (see, e. g., [27]), the number density is typically lower ($n \sim 10^{18} \text{ cm}^{-3}$) or even $\sim 10^{16} \text{ cm}^{-3}$ if CO_2 laser is used for heating. In such conditions, even impurities are usually strongly magnetized and $q \geq 1$. Nevertheless, the current-driven diffusion is still rather efficient and the diffusion velocity V_B (Eq. (16)) reaches the values $\sim 10^4\text{--}$

10^5 cm/s. Correspondingly, the chemical structures can be generated in such plasma on a timescale of milliseconds. The considered diffusion mechanism can also operate in plasma of θ -pinch. Such configurations are very suitable to study diffusion processes because of their long lifetime. Typical number density and temperature are $\sim 10^{18}$ cm $^{-3}$ and 10^7 – 10^8 K, respectively. Plasma is essentially magnetized in θ -pinch since $q \sim 10^3$ – 10^4 (see, e. g., [1]) but, nevertheless, there is enough time for generation of chemical spots because of sufficiently long lifetime.

Our study reveals that a particular type of waves may exist in multicomponent plasma in the presence of electric currents. These waves are slowly decaying and characterized by oscillations of the impurity number density alone. They exist only if the magnetic field is so strong that the magnetic pressure is greater than the gas pressure. Generally, the frequency of such waves turns out to be different for different impurities. This frequency is rather low and is determined mainly by a diffusion timescale. If $M = 0$, it can be estimated as $\omega_I \sim kD_B/L \sim c_A^2 \tau_i / A_i L \lambda$ where $\lambda = 2\pi/k$ is the wavelength of waves. In astrophysical conditions, such waves can manifest themselves in the atmospheres of magnetic stars where the magnetic field is of the order of 10^4 G and the number density and temperature are 10^{14} cm $^{-3}$ and 10^4 K, respectively. If the lengthscale, L , and the wavelength, λ , are of the same order of magnitude (for instance, $\sim 10^{11}$ cm), then the period of compositional waves is $\sim 3 \times 10^3$ yrs. This is much shorter than the stellar lifetime and generation of such waves in the atmospheres should lead to spectral variability with the corresponding timescale.

Compositional waves can occur in laboratory plasmas as well but their frequency is essentially higher. If $B \sim 10^5$ G, $n \sim 10^{15}$ cm $^{-3}$, $T \sim 10^6$ K, and $L \sim \lambda \sim 10^2$ cm, then the period of compositional waves is $\sim 10^{-8}$ s. Note that this is only the order of magnitude estimate but frequencies of various impurities can differ essentially since the period of compositional waves depends on the sort of heavy ions. In terrestrial conditions, the compositional waves also can manifest themselves by oscillations in spectra. Note that these waves exist only if the magnetic pressure is greater than the gas pressure. The current-driven diffusion can be important not only in plasma but in some conductive fluids if the magnetic field is sufficiently strong there.

The author thanks the Russian Academy of Sciences for financial support under the program OFN-15.

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